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Quasi-fixed points and charge and colour breaking in low scale models

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Abstract: We show that the current LEP2 lower bound upon the minimal supersymmetric standard model (MSSM) lightest Higgs mass rules out quasi-fixed scenarios for string scales between 10^6 and 10^{11} GeV unless the heaviest stop mass is more than 2 TeV. We consider the implications of the low string scale for charge and colour breaking (CCB) bounds in the MSSM, and demonstrate that CCB bounds from F and D-flat directions are significantly weakened. For scales less than 10^{10} GeV these bounds become merely that degenerate scalar mass squared values are positive at the string scale.

Keywords: Supersymmetric Standard Model, Supersymmetry Breaking, Superstring Vacua.

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1. Introduction

For many years, string and unification scales were thought to be high ($\gtrsim 10^{16}\,\mathrm{GeV}$). The perturbative heterotic formulation of string theory had the fundamental string scale $\Lambda_s \sim O(10^{17})\,\mathrm{GeV}$ close to $M_{\mathrm{Planck}} \sim 10^{19}\,\mathrm{GeV}$ because of its constrained description of the gravitational interaction. The grand unification (GUT) scale was around $\Lambda_{\mathrm{GUT}} \sim 10^{16}\,\mathrm{GeV}$, motivated by the apparent convergence of the gauge couplings when they were evolved to this value. Recently however, attention has turned to models that have lower string and/or unification scales [1]–[6] and this has raised some interesting questions to do with renormalisation group evolution of parameters.

The most immediate is of course whether gauge or Yukawa unification is still possible or even necessary with a lower string scale. One example that achieves gauge unification at the string scale [2] has the couplings experience power law "running" [2, 4, 5] above a compactification scale due to the presence of additional Kaluza-Klein modes. A Kaluza-Klein spectrum with the same ratios of gauge beta functions as those in the MSSM leads to a logarithmic running up to the compactification scale with rapid power law unification taking place very rapidly thereafter [2]. An example that does not achieve gauge unification is "mirage" unification [6]. In mirage unification the gauge couplings at the string scale receive moduli dependent corrections that behave as if there were continued logarithmic running above the string scale up to unification at the usual Λ_{GUT}. "Mirage unification" refers to this fictitious unification.[1]

[1] Note that although there are problems with the particular string realisation of mirage unification in ref. [6], the idea may be realisable in other models and remains an interesting possibility.

A particularly attractive choice for the string scale (albeit one that is not immediately accessible to experiment) is $\Lambda_s \sim 10^{11}$ GeV [3]. In this case the hierarchy between the weak scale and the Planck scale arises without unnaturally small ratios of fundamental scales. It was also noted in the first reference of [3] that $\Lambda_s \sim 10^{11}$ GeV gives neutrino masses of the right order. We return to this model below and refer to it as the Weak-Planck (WP) model.

In this paper we consider two other related issues in the Minimal Supersymmetric Standard Model (MSSM),

$$W_{\mathrm{MSSM}} = h_U Q H_2 U^c + h_D Q H_1 D^c + h_E L H_1 E^c + \mu H_1 H_2\,, \tag{1.1}$$

with a low string scale. The first concerns the top quark Quasi-Fixed Point (QFP). The QFP is characterised by a focusing of some MSSM parameters to particular ratios as the renormalisation scale Λ is decreased towards the top quark mass, m_t [7, 10, 12]. Formally it is defined to be the point in parameter space where there is a Landau pole in the top Yukawa coupling h_t at the string or GUT scale (whichever is the lower). In practice however this focusing behaviour can occur for a large but finite $h_t(\Lambda_s)$, still treat-able by perturbation theory. The coupling h_t focusses to some value at m_t independent of $h_t(\Lambda_s)$ provided it is large enough. In low scale models, with their foreshortened logarithmic running, one naturally expects this behaviour to be very different. If the pole is at $\Lambda_s < \Lambda_{\mathrm{GUT}}$, we expect the quasi-fixed value of the top Yukawa at m_t to be larger than for the usual GUT scale unification. Conversely, for a given value of top mass and $\tan\beta$ at the weak scale the model will be further from the QFP for $\Lambda_s < \Lambda_{\mathrm{GUT}}$. We shall determine the QFP prediction for $h_t(m_t)$, on which experimental constraints from LEP2 can be brought to bear in order to empirically constrain Λ_s assuming the QFP scenario. In particular, we consider the empirically derived lower bound upon the lightest CP-even MSSM Higgs mass, which in the canonical GUT scenarios has been shown to be a strong restriction upon the QFP scenario [12].

The second issue we consider is the possibility of minima that break charge and colour lying along F and D-flat directions [10, 13, 14, 15, 16, 17, 18]. The constraints found by requiring that there be no such (CCB) minima are dependent on the distance from the QFP. They are most severe at the QFP itself [10, 15, 16] and indeed, in the usual MSSM at the QFP, CCB constraints exclude half the parameter space. With a lower string scale it seems likely that such constraints will generally be less restrictive for two reasons. First, a given point in (weak-scale) parameter space will be further from the QFP as noted above. Second, the CCB minima are generated radiatively when the mass-squared parameter for H_2 becomes negative. When there is a lower string scale there is less "room" for a minimum to form at vacuum expectation values (VEVs) much greater than the weak scale. (More specifically, there are positive mass-squared contributions to the potential along the flat direction that become dominant at lower VEVs.) We shall

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demonstrate that this is indeed the case and that for $\Lambda_s \lesssim 10^{10}$ the CCB constraint (at least along the F and D-flat directions) is merely that scalar mass squared values are positive.

We will throughout be discussing these aspects by assuming that there is the standard logarithmic running of the MSSM upto a scale, Λ_s, that we rather loosely refer to as the string scale. This scale may be much lower than $\Lambda_{\rm GUT}$. We define the QFP to be where the top Yukawa has a Landau pole at this point, since any variation in the Yukawa couplings above Λ_s is expected to be drastically changed by string physics. As for the CCB bounds, we derive them on the soft breaking parameters at Λ_s since this is close to the scale at which we expect the supersymmetry breaking parameters to be derived in any fundamental string model (although we will have more to say on this in due course).

2. The quasi-fixed MSSM

The QFP [7, 10, 12] constraint, i.e. that the top Yukawa coupling h_t has a Landau pole at the string scale, gives important predictions in terms of the couplings and masses of supersymmetric particles [7, 10, 12]. We now examine the prediction for $h_t(m_t)$ numerically, paying special attention to its dependence on the string scale. Fermion masses and gauge couplings are set to be at their central values in ref. [19] except for $\alpha_s(M_Z)$, which is varied to show the induced uncertainty. Below m_t, we run using a three-loop QCD $\otimes$ one-loop QED effective theory with all superpartners integrated out.

In order to illustrate the quasi-fixed behaviour we first make a rough calculation. To this end, we approximate the superparticle spectrum to be degenerate at m_t, allowing us to use the (two-loop) MSSM renormalisation group equations above that scale. Figure 1 illustrates the quasi-fixed behaviour for two values of string scale. The dependence of the low scale h_t on its string scale value is shown for canonical QFP SUSY GUT framework with string/unification scale $\Lambda_s = 2 \times 10^{16}$ GeV. The almost horizontal part of the lines represent the QFP regime: where, for input values $h_t(\Lambda_s) > 1.5$,

$$h_t(m_t) = 1.10 \pm 0.02 \tag{2.1}$$

results. Lowering Λ_s to 10^{11} GeV, as in the WP model, we see that the quasi-fixed behaviour is diminished somewhat, as indicated by the more positive slope of the relevant lines. However, for $h_t(\Lambda_s) > 1.5$ a QFP value of

$$h_t(m_t) = 1.17 \pm 0.04 \tag{2.2}$$

occurs.[2]

[2]Errors quoted here include those due to the error in $\alpha_s(M_Z)$ but they do not include those from non-degeneracy in the superparticle spectrum.

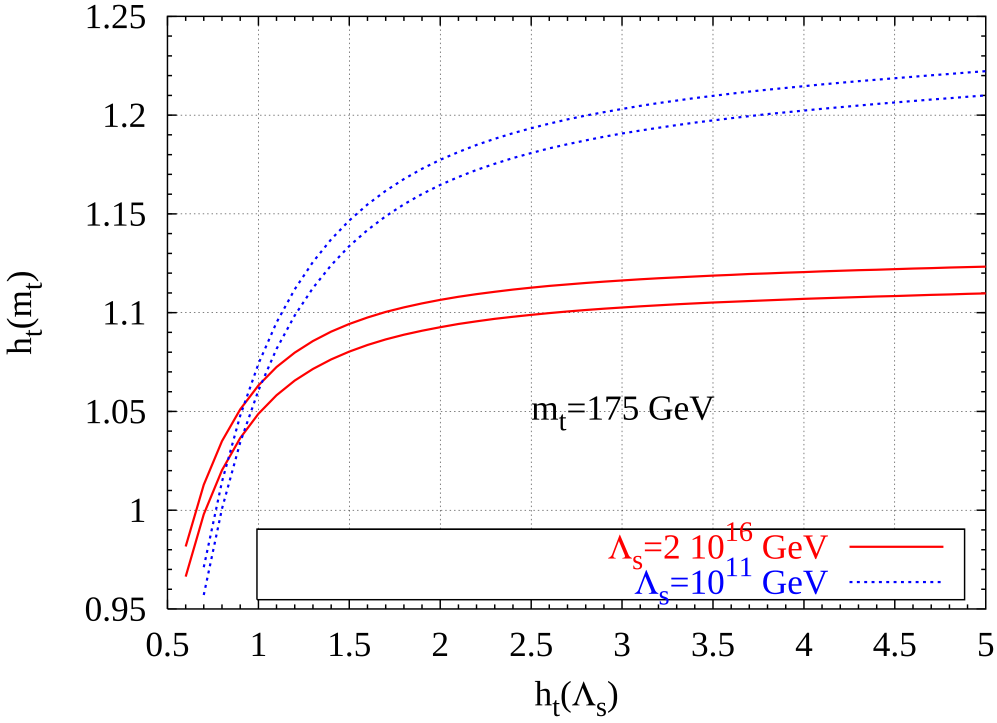

Figure 1: Prediction of low energy top Yukawa coupling $h_t(m_t)$ for string scale input $h_t(\Lambda_s)$. Two string scales $\Lambda_s = 10^{11}, 2 \times 10^{16}$ GeV are used. The pair of lines represent the range produced by varying $\alpha_s(M_Z) = 0.115 - 0.122$ (the upper lines corresponding to higher $\alpha_s(M_Z)$).

The $h_t(m_t)$ QFP prediction can be turned into a prediction of the MSSM parameter $\tan\beta$ (the ratio of the two neutral Higgs VEVs) through the relation

$$\sin\beta = \frac{\sqrt{2} m_t(m_t)}{v h_t(m_t)} \tag{2.3}$$

and the known value [19] of the top quark mass, $m_t = 175 \pm 5$ GeV. We obtain the running top mass $m_t(m_t)$ from m_t by employing the one-loop QCD correction, thus assuming that supersymmetric corrections to it are small. v refers to the Standard Model Higgs VEV of 246.22 GeV. Low values of $1 < \tan\beta < 3$ result from eq. (2.3) when a quasi-fixed value $h_t(m_t) > 1.05$ is used. The range of $\tan\beta$ relevant here is constrained by the non-observation of the lightest MSSM Higgs boson at LEP2 [12]. The current limits [20] exclude $m_{h^0} < 107.7$ GeV for the low $\tan\beta < 3$ scenario. Quasi-fixed $\tan\beta$ predictions are illustrated in table 1, where they are displayed with estimated uncertainties for the WP and GUT quasi-fixed scenarios. The uncertainties are induced by those quoted in the $h_t(m_t)$ predictions in eqs. (2.1) and (2.2).

Here, we set $h_t(\Lambda_s) = 5$, close to its Landau pole and near the edge of perturbativity. In ref. [21], the limit $h_t < 3$ was used to define a perturbative regime and we will use the point $h_t(\Lambda_s) = 3$ as an estimator of sensitivity to $h_t(\Lambda_s)$. A central value of $\alpha_s(M_Z) = 0.119$ [19] was used. We display the results for $m_t = 170, 175, 180\,\mathrm{GeV}$ to illustrate the large dependence upon the top mass. We use the two-loop diagrammatic result in ref. [22] to calculate the MSSM lightest Higgs mass with the state-of-the-art program `FeynHiggsFast2`. Corrections to the values of $h_t(m_t)$ displayed in figure 1 from including sparticle thresholds are expected to be small because the majority of change in $h_t(\mu)$ occurs in the running between Λ_s and $1000\,\mathrm{GeV}$, identical in both cases. We therefore use the prediction for $h_t(m_t)$ as calculated with a degenerate sparticle spectrum at m_t. To within small errors, this value should still be applicable for a non-degenerate spectrum, which is what we assume here.

Λ_s (GeV)	$m_t = 170\,\mathrm{GeV}$	$m_t = 180\,\mathrm{GeV}$
10^{11}	$1.27^{+0.13}_{-0.11}$	$1.51^{+0.20}_{-0.14}$
$2 \cdot 10^{16}$	$1.52^{+0.10}_{-0.09}$	$1.93^{+0.08}_{-0.15}$

Table 1: $\tan\beta$ prediction for a top-Yukawa QFP at the GUT scale or the WP scale.

Ideally, we would now perform a parameter scan through the low energy supersymmetry breaking parameters in order to determine the maximum value of m_{h^0} consistent with the QFP. This is impractical however, and we resort to using a benchmark point in low energy supersymmetry breaking parameter space. The value of m_{h^0} obtained by the benchmark corresponds in practice to be very close (within one GeV) to a more general upper bound on m_{h^0} [22], given an upper bound on sparticle masses. For generality, this benchmark corresponds to *non-universal* SUSY breaking parameters. For a given value of Λ_s, $\tan\beta$ is predicted by the QFP as in figure 1. We then set μ and the parameter

$$X_t \equiv A_t - \mu \cot\beta = 2m_{\tilde{t}_2}\,. \tag{2.4}$$

As is argued in [22], $X_t \approx 2m_{\tilde{t}_2}$ corresponds to the maximal-mixing case, where m_{h^0} is maximised. A_t is then specified by eq. (2.4), and therefore the gluino mass will be set by the QFP prediction of A_t/M_3. For $\Lambda_s = 2 \times 10^{16}\,\mathrm{GeV}$ for example, we obtain $A_t/M_3 = -0.59$ [10]. However to the order in perturbation theory used here, the Higgs mass is independent of the gluino mass. Fixing M_A then sets B through the relation [11]

$$M_A^2 = \frac{2\mu B}{\sin 2\beta}\,. \tag{2.5}$$

The two electroweak symmetry breaking conditions are [11]

$$\bar{m}_1^2 + \bar{m}_2^2 = -M_A^2\,, \qquad \tan^2\beta\left(\bar{m}_2^2 + M_Z^2/2\right) = \bar{m}_1^2 + M_Z^2/2\,, \tag{2.6}$$

where $\bar{m}_i^2 = m_{H_i}^2 + \mu^2$ plus loop corrections. Together, they determine the Higgs mass soft breaking parameters $m_{H_1}^2$ and $m_{H_2}^2$ (conservatively assumed to be uncorrelated and free). Following the authors of ref. [22], the maximum value of m_{h^0} is assumed to be acquired by taking[3] $M_2 = 100\,\mathrm{GeV}$, $M_A = 1000\,\mathrm{GeV}$, $\mu = -100\,\mathrm{GeV}$ and $m_{\tilde{t}_2} =$

[3]See ref. [22] for a definition of these parameters.

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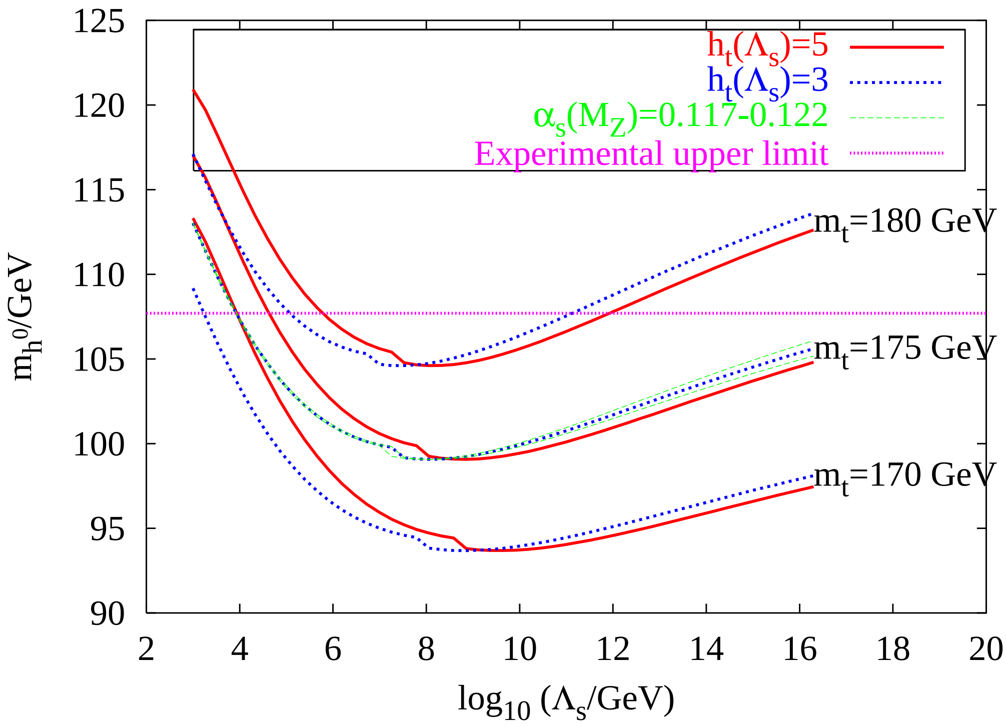

Figure 2: Theoretical upper bound on lightest MSSM Higgs mass in the quasi-fixed scenario with varying string scale Λ_s. Bounds for quasi-fixed top Yukawa couplings $h_t(\Lambda_s) = 3, 5$ and $\alpha_s(M_Z) = 0.119$ are shown. The copies of each curve are for $m_t = 180, 175, 170$ GeV from top to bottom, respectively. For $m_t = 175$ GeV and $h_t(\Lambda_s) = 3$, we have displayed the variation due to the error on $\alpha_s(M_Z) = 0.117 - 0.122$ via the lighter dashed curves. The area underneath the experimental limit has been excluded for the MSSM by LEP2. See text for a description of the other MSSM parameters used.

2000 GeV in order to get a conservative estimate. Dependence of the upper bound on m_{h^0} is logarithmic in this parameter and therefore slowly increasing as $m_{\tilde{t}_2}$ increases. Therefore, to obtain a sizeable effect on the bound, unnaturally high values of $m_{\tilde{t}_2}$ would have to be taken. Using the above procedure, the soft breaking parameters that m_{h^0} depends most sensitively upon are fixed near the weak scale without reference to any further unification assumptions, such as minimal supergravity for example.

Figure 2 displays the QFP value of m_{h^0} predicted by the benchmark by varying Λ_s. Uncertainties induced by the 1σ error on $\alpha_s(M_Z)$ are shown for one particular case. It is larger for higher Λ_s, but always less than 0.5 GeV and much smaller than the uncertainty induced by the empirical error on m_t. In fact, we see from the figure that the QFP is ruled out to better than 1σ for the range

$$10^6 < \frac{\Lambda_s}{\text{GeV}} < 10^{11} \tag{2.7}$$

for $m_t = 170-180$ GeV and $h_t(\Lambda_s) = 3-5$. If we take $m_t = 175$ GeV, the QFP is ruled out for any $\Lambda_s > 10^5$ GeV. As noted above, the $h_t(\Lambda_s) = 3$ curves give an estimate of the uncertainty in the QFP prediction. The figure shows that this dependence is small for $\Lambda_s > 10^9\,\mathrm{GeV}$ but that it increases for $\Lambda_s < 10^9\,\mathrm{GeV}$. However, we note that in this latter range, h_t being less than 5 (but still in the quasi-fixed regime) actually *strengthens* the upper bound upon m_{h^0}. $h_t(\Lambda_s) = 5$ thus gives a reasonably accurate bound for $\Lambda_s > 10^9\,\mathrm{GeV}$ and a conservative one for $\Lambda_s < 10^9\,\mathrm{GeV}$.

3. Analytic CCB bounds at low string scales

We now turn to the discussion of CCB bounds. Unphysical CCB minima present some of the most severe bounds for supersymmetric models [10], [13]–[18]. Indeed, for a number of models it has been found that they exclude much of the parameter space not already excluded by experiment; for example the MSSM where supersymmetry breaking is driven by the dilaton [14], SUSY GUTS at the low $\tan\beta$ quasi-fixed point (QFP) [10], M-theory in which supersymmetry breaking is driven by bulk moduli fields [16, 17] and several other string/field theory scenarios [17, 18]. All of the above work, however, assumed a logarithmic evolution of the gauge couplings with unification at a high scale $\geq 10^{16}\,\mathrm{GeV}$.

In this section we shall be considering the effect of truncating this logarithmic evolution at a low string scale. For completeness, we first recall the three types of CCB minima that can occur in supersymmetric models:

- D-flat directions which develop a minimum due to large trilinear supersymmetry breaking terms.
- F and D-flat directions corresponding to a single gauge invariant.
- F and D-flat directions which correspond to a combination of gauge invariants [24] involving H_2 [23].

Since the first type are important at low scales [13] and the second type are only important when there are negative mass-squared terms at the GUT scale, we shall concentrate on the constraints coming from the last type of minimum. These occur at intermediate scales due to the running H_2 mass-squared even if all the mass-squared values are positive at the GUT scale. Hence the resulting constraints are very dependent on renormalisation group running at *high* scales and are particularly interesting from the point of view of models with a lower string scale. As discussed above, our initial expectation is that the CCB bounds will be far less severe than in the usual versions of the MSSM.

We will consider the F and D-flat direction in the MSSM corresponding to the operators

$$L_i L_3 E_3\,; \qquad H_2 L_i\,, \tag{3.1}$$

where the suffices on matter superfields are generation indices. With the following choice of VEVs;

$$h_2^0 = -\frac{a^2\mu}{h_{E_{33}}}\,, \qquad \tilde{e}_{L_3} = \tilde{e}_{R_3} = \frac{a\mu}{h_{E_{33}}}\,, \qquad \tilde{\nu}_i = \frac{a\sqrt{1+a^2}\mu}{h_{E_{33}}}\,, \tag{3.2}$$

the potential along this direction depends only on the soft supersymmetry breaking terms (neglecting a small D-term contribution);

$$V = \frac{\mu^2}{h_{E_{33}}^2} a^2 \left(a^2 \left(m_2^2 + m_{L_{ii}}^2 \right) + m_{L_{ii}}^2 + m_{L_{33}}^2 + m_{E_{33}}^2 \right). \tag{3.3}$$

In the usual MSSM we can reasonably assume that, since the CCB minimum forms at VEVs corresponding to $a \gg 1$, the largest relevant mass, and therefore the appropriate scale to evaluate the parameters at, is $\phi = h_{U_{33}}\langle h_2^0\rangle \equiv h_t\langle h_2^0\rangle$. This minimises the top quark contributions to the effective potential at one-loop. Further corrections to the potential are assumed to be small. Once we lower the string scale however we encounter the problem that the CCB minimum moves towards low scales and that consequently this approximation breaks down. Evidently, from eq. (3.3), this happens precisely where the positive $m_{L_{ii}}^2 + m_{L_{33}}^2 + m_E^2$ terms begin to dominate, and so we do not anticipate that CCB minima will be formed when $a < 1$. In order to check this however, our approach will be to construct the constraints using the above assumption on ϕ and observe that they get far less restrictive as we move to moderately low string scales, say $\Lambda_s \sim 10^8$ GeV. We then check the approximate one-loop analytic results obtained with a more accurate two-loop numerical analysis at certain parameter points and observe numerically that CCB minima do not reappear as we move to very low string scales where $a < 1$.

In the above potentials, $\langle h_2^0\rangle = -a^2\mu/h_{E_{33}}$ so that the eq. (3.3) is of the form

$$V = \frac{\Lambda^2}{h_{U_{33}}^2}\hat{\phi}\left(\hat{\phi}A + \frac{B}{b}\right), \tag{3.4}$$

where $A = m_2^2(\phi) + m_{L_{ii}}^2(\phi)$, B is the LLE combination of mass-squared parameters (also evaluated at ϕ) that appears in the potential,

$$\hat{\phi} = \frac{\phi}{\Lambda} \tag{3.5}$$

and Λ is an arbitrary scale which we shall take to be the usual unification scale $\Lambda_{\mathrm{GUT}} \sim 10^{16}$ GeV. The bound is therefore governed by A, B and the parameter

$$b(\phi) = \frac{\Lambda_{\mathrm{GUT}} h_{E_{33}}}{h_{U_{33}}\mu} \tag{3.6}$$

for the LLE, LH_2 direction described above, or

$$b(\phi) = \frac{\Lambda_{\mathrm{GUT}} h_{D_{33}}}{h_{U_{33}}\mu} \tag{3.7}$$

for the equally dangerous LQD, LH_2 direction.

To estimate the bound, we now adapt the results of refs. [15, 16]. At large values of $a \gg 1$ the potential is governed by the first term. Whatever the string scale may be, we require that m_2^2 be positive there and negative at M_W (for successful electroweak symmetry breaking). A CCB minimum radiatively forms close to the value ϕ_p where A first becomes negative (typically at a scale of $few \times \mu/h_{E_{33}}$) [15, 16].

In refs. [15, 16] it was shown that once we are able to estimate ϕ_p the bound follows fairly easily, and this was done for models with degenerate gaugino masses. Bounds were derived for all non-universal scalar masses and couplings. In the present case however, the gauge couplings and the gaugino masses are *also* non degenerate at the string scale Λ_s.

This makes a general analytic treatment of the RGEs extremely difficult, so in order to simplify matters we shall henceforth assume the "GUT gaugino relation". That is we assume that at the scale Λ_s we have the usual GUT expression for gaugino masses,

$$\frac{M_a}{M_b} = \frac{\alpha_a}{\alpha_b}. \tag{3.8}$$

This relationship has the useful property that the gaugino masses as well as the gauge couplings would be degenerate if we continued the evolution of the MSSM RGEs upto $\Lambda_{\rm GUT}$. We shall call this fictitious degenerate value $M_a(\Lambda_{\rm GUT}) = M_{1/2}$. Note that eq. (3.8) is only valid to one-loop order, and indeed in this section we present analytic results to one-loop order only (contrary to the last section).

Although eq. (3.8) may seem like a rather brutal requirement, it holds for a number of interesting cases, for instance in models with power law unification as shown in ref. [5]. In these models the scale Λ_s in our analysis should really be interpreted as the *compactification* scale at which the first Kaluza-Klein states appear in the spectrum, rather than the string scale which is where we expect the real gauge unification to take place after a short period of power law "running". An assumption such as degenerate soft terms at the compactification scale Λ_s is consistent with, for example, the Scherk-Schwarz mechanism of supersymmetry breaking.

Equation (3.8) is also expected to hold in the mirage unification models of ref. [6] when there is no S/T-mixing and in the limit $T + \overline{T} \to \infty$. In this limit we have

$$M_a \approx \sqrt{3} m_{3/2} \sin\theta \frac{\alpha_a}{\alpha_0} + \mathcal{O}\left(1/(T+\overline{T})^2\right), \tag{3.9}$$

where we use the subscript-0 to represent values at the usual $\Lambda_{\rm GUT}$ unification scale (i.e. $\alpha_0 \approx 1/25$), and where we have neglected terms of order $\alpha_a m_{3/2}$ which is consistent to one-loop accuracy. In this case we have $M_{1/2} = \sqrt{3} m_{3/2} \sin\theta$.

Equation (3.8) allows us to adapt the expressions of ref. [15] with only a modest amount of effort by writing the parameters at Λ_s in terms of their values at $\Lambda_{\rm GUT}$. In order to proceed, we next spend a little time discussing the analytic solutions to the renormalisation group running. The solutions of all the parameters may easily

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be expressed in terms of those combinations with infrared QFPs; $R = h_t^2/g_3^2$, A_t and $3M^2 = m_2^2 + m_{U_{33}}^2 + m_{Q_{33}}^2$. These may be written as functions of

$$r = \frac{\alpha_0}{\alpha_3} \equiv \frac{1}{\tilde{\alpha}_3} = 1 + \frac{6\alpha_0}{4\pi} \log \frac{\Lambda}{\Lambda_{\rm GUT}} \,, \tag{3.10}$$

so that

$$\frac{\alpha_0}{\alpha_2} \equiv \frac{1}{\tilde{\alpha}_2} = \frac{3}{4-r} \,; \qquad \frac{\alpha_0}{\alpha_1} \equiv \frac{1}{\tilde{\alpha}_1} = \frac{5}{16-11r} \,. \tag{3.11}$$

Taking $\alpha_3(m_t) = 0.108$ means that $0.37 < r < 1$ with $r = 1$ corresponding to the GUT scale. If the string scale is at $\Lambda_s = 10^{11}\,\mathrm{GeV}$ as in the WP model, then the corresponding value of $r_s \equiv r(\Lambda_s)$ is $r_s = 0.82$. It is useful to define

$$\Pi(r) = \tilde{\alpha}_3^{16/9} \tilde{\alpha}_2^{-3} \tilde{\alpha}_1^{-13/99} \,, \qquad \hat{J} = \frac{1}{r\Pi(r)} \int_r^1 \Pi(r')\, dr' \,. \tag{3.12}$$

Solving for R in terms of its value R_s at the string scale (we use subscript-s to denote string-scale values) we find

$$\frac{1}{R} = \frac{\Pi_s r_s}{R_s \Pi r} + \frac{1}{R^{\rm QFP}} \,, \tag{3.13}$$

where the QFP value (where the Yukawa couplings blow up at the string scale) is given by

$$\frac{1}{R^{\rm QFP}} = 2\hat{J}(r) - 2\hat{J}(r_s) \frac{\Pi_s r_s}{\Pi r} \,. \tag{3.14}$$

We also, for later use, define the distance from the real QFP,

$$\sigma = \frac{R}{R^{\rm QFP}} \,. \tag{3.15}$$

This can be rewritten in terms of a fictitious renormalisation of R down from a $\Lambda_{\rm GUT}$ scale value of R_0, i.e. defining

$$\frac{1}{\overline{R}^{\rm QFP}} = 2\hat{J} \tag{3.16}$$

we have

$$\frac{1}{R} = \frac{1}{R_0 \Pi r} + \frac{1}{\overline{R}^{\rm QFP}} \,, \qquad \frac{1}{R_s} = \frac{1}{R_0 \Pi_s r_s} + \frac{1}{\overline{R}_s^{\rm QFP}} \,. \tag{3.17}$$

This is the usual expression for R (c.f. ref. [16]); however it should be noted that R_0 is here merely a parameter that is negative in the region $1/\overline{R}^{\rm QFP} > 1/R_s > 0$. In the usual MSSM with unification at the GUT scale, this would of course be an unphysical (non-perturbative) region. For A_t and M^2 we now define the distance from the usual QFP (i.e. where couplings blow up at the usual unification scale $\Lambda_{\rm GUT}$)

$$\rho = \frac{R}{\overline{R}^{\rm QFP}} \tag{3.18}$$

and also

$$\xi = \frac{1-r}{r\hat{J}} - 1\,. \tag{3.19}$$

We then obtain expressions for $\tilde{A}_t = A_t/M_{1/2}$ and $\tilde{M}^2 = M^2/M_{1/2}^2$ in terms of their fictitious values, $\tilde{A}_0$ and $\tilde{M}_0^2$, at Λ_{GUT};

$$\tilde{A}_t = (1-\rho)\tilde{A}_0 + \rho\xi - \Gamma\,, \qquad \tilde{M}^2 = (1-\rho)\tilde{M}_0^2 - \frac{1}{3}\rho\tilde{K} + \frac{2}{3}(1-r)\gamma\,, \tag{3.20}$$

where

$$\begin{aligned}
\gamma &= \frac{16}{9}\tilde{\alpha}_3\left(1+\tilde{\alpha}_3\frac{(1-r)}{2}\right) + \tilde{\alpha}_2\left(1-\tilde{\alpha}_2\frac{(1-r)}{6}\right) + \\
&\quad + \frac{13}{45}\tilde{\alpha}_1\left(1-11\tilde{\alpha}_1\frac{(1-r)}{10}\right),\\
\Gamma &= (1-r)\left(\frac{16}{9}\tilde{\alpha}_3 + \tilde{\alpha}_2 + \frac{13}{45}\tilde{\alpha}_1\right),\\
\tilde{K} &= (1-\rho)(\xi-\tilde{A}_0)^2 - \xi^2 + (\xi+1)\Gamma\,,\\
\tilde{A}_0 &= \frac{\left(\tilde{A}_s - \rho_s\xi_s + \Gamma_s\right)}{(1-\rho_s)}\,,\\
\tilde{M}_0^2 &= \frac{\left(\tilde{M}_s^2 + \frac{1}{3}\rho_s\tilde{K}_s - \frac{2}{3}(1-r_s)\gamma_s\right)}{(1-\rho_s)}\,.
\end{aligned} \tag{3.21}$$

It is important to note that, since

$$1-\rho = (1-\sigma)(1-\rho_s)\,, \tag{3.22}$$

A_t and M^2 retain their QFP behaviour since when $\sigma = 1$ (or $R_s \to \infty$) they are both independent of their values *at the string scale*, Λ_s. In addition, factors of $1/(1-\rho_s)$ cancel so that there is no divergent behaviour at the usual QFP. Also note that this QFP is at lower $\tan\beta$ than in the usual MSSM unification. We can estimate the difference in $\tan\beta$ at the QFP by using

$$R = \frac{m_t^2}{4\pi\alpha_3 v^2 \sin^2\beta}\,, \tag{3.23}$$

so that

$$\sin^2\beta^{\mathrm{QFP}} = \frac{\overline{R}^{\mathrm{QFP}}}{R^{\mathrm{QFP}}}\sin^2\overline{\beta}^{\mathrm{QFP}}\,. \tag{3.24}$$

Equations (3.14) and (3.16) then give $\tan\beta^{\mathrm{QFP}} \approx 1.2$ in the WP model with $\Lambda_s = 10^{11}$ GeV, in agreement with the full two-loop numerical result presented in figure 1.

With all parameters expressed in terms of GUT scale parameters, we are now simply able to apply the bounds derived in ref. [16] for non-universal SUSY breaking

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directly. Consider for example the LH_2, LLE direction. The cosmological bounds in this case are

$$\left(2\tilde{m}^2_{L_{ii}} + \tilde{m}^2_2 - \tilde{m}^2_{U_{33}} - \tilde{m}^2_{Q_{33}}\right)\Big|_0 \gtrsim f(\tilde{B}|_0) + (\rho_p - 1) \times \\ \times \left(g(\tilde{B}|_0) + 3\tilde{M}^2|_0 - \rho_p\left(1 - \tilde{A}_0\right)^2\right), \tag{3.25}$$

where ρ_p is the value of ρ at the scale ϕ_p and

$$\begin{aligned} f(x) &= 1.20 - 0.14x + 0.02x^2\,, \\ g(x) &= 2.77 - 0.18x + 0.02x^2\,, \\ B &= m^2_{L_{ii}} + m^2_{L_{33}} + m^2_{E_{33}}\,, \end{aligned} \tag{3.26}$$

for $\mu = 500\,\mathrm{GeV}$. (The small dependence of f and g on μ, which we must choose by hand, is discussed in ref. [16].) To a good approximation the value of ρ_p is given by [16]

$$\frac{1}{\rho_p} = 1 + \frac{1}{2R_0} = 1 + 3.17\left(\sin^2\beta - \sin^2\overline{\beta}^{\mathrm{QFP}}\right). \tag{3.27}$$

In order to relate the quantities to their string scale values, we use the one-loop RGE solutions for A and B;

$$\begin{aligned} &\left(2\tilde{m}^2_{L_{ii}} + \tilde{m}^2_2 - \tilde{m}^2_{U_{33}} - \tilde{m}^2_{Q_{33}}\right)\Big|_s \gtrsim \\ &\quad\gtrsim -\frac{16}{9}\delta^{(2)}_{3s} - 3\delta^{(2)}_{2s} - \frac{5}{99}\delta^{(2)}_{1s} + f(\tilde{B}|_0) + \\ &\qquad + (\sigma_p - 1)\Big\{(1 - \rho_s)g(\tilde{B}|_0) + 3\tilde{M}^2|_s - 2(1 - r_s)\gamma_s + \\ &\qquad\qquad + \rho_s\Big(-1 + \rho_s(\xi_s - 1)^2 - \Gamma_s(\xi - 3) - 2\tilde{A}_s(\xi - 1)\Big) - \\ &\qquad\qquad - \sigma_p(\tilde{A}_s - \rho_s\xi_s + \Gamma_s + \rho_s - 1)^2\Big\}, \end{aligned} \tag{3.28}$$

where

$$\delta^{(n)}_i = \frac{\alpha^n_i}{\alpha^n_0} - 1\,, \qquad \tilde{B}\Big|_s = \tilde{B}\Big|_0 - 3\delta^{(2)}_{2s} - \frac{1}{11}\delta^{(2)}_{1s}\,, \tag{3.29}$$

and where

$$\sigma_p = 1 - \frac{\Pi_s r_x}{\Pi_s r_s(1 - \rho_s) + 2R_s}\,. \tag{3.30}$$

The general behaviour of the bounds is clearly similar to that in the usual unification scenario. The bounds are on the particular combination $(2\tilde{m}^2_{L_{ii}} + \tilde{m}^2_2 - \tilde{m}^2_{U_{33}} - \tilde{m}^2_{Q_{33}})|_s$ and are most restrictive at the QFP, decreasing as $\tan\beta$ increases. Away from the QFP there is a quadratic dependence on $\tilde{A}_s$ with a minimum at $\tilde{A}_s = \mathcal{O}(1)$.

We can now see why the bounds at low scales are far less severe than in the MSSM with unification at the GUT scale. First, close to the QFP, the bound is

$$\begin{aligned} \left(2\tilde{m}^2_{L_{ii}} + \tilde{m}^2_2 - \tilde{m}^2_{U_{33}} - \tilde{m}^2_{Q_{33}}\right)\Big|_s &\gtrsim -\frac{16}{9}\delta^{(2)}_{3s} - 3\delta^{(2)}_{2s} - \frac{5}{99}\delta^{(2)}_{1s} + f(\tilde{B}|_0) \\ &= -0.48 + f(\tilde{B}|_0)\,, \end{aligned} \tag{3.31}$$

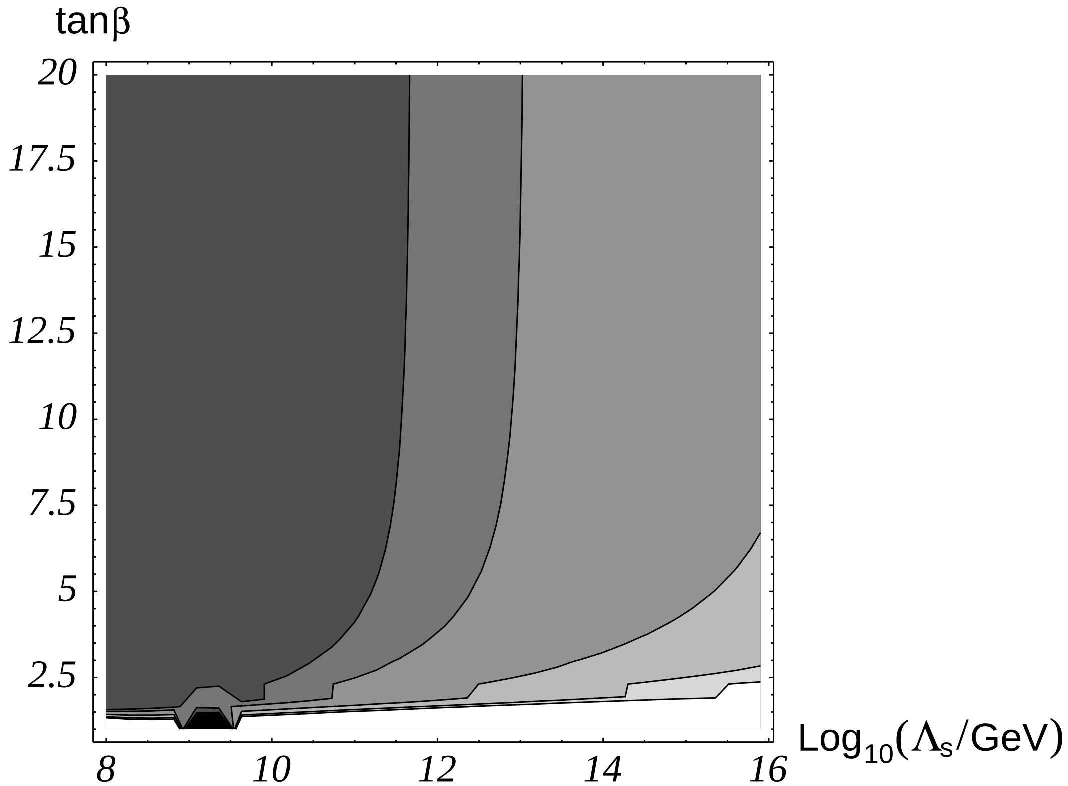

Figure 3: Charge and colour breaking bounds with a lower string scale, Λ_s, for $\mu =$ 500 GeV and degenerate trilinear terms, $A = -M_{1/2}$, and scalar masses m_s, at the string scale. The figure shows bounds on $\tilde{m}_s^2 = m_s^2/M_{1/2}^2$ for varying Λ_s and $\tan\beta$ (i.e. away from the QFP). The contours are $\tilde{m}_s^2 > 0$ (black), $\tilde{m}_s^2 > 0.25$ (medium dark), $\tilde{m}_s^2 > 0.33$ (medium), $\tilde{m}_s^2 > 0.5$ (medium light), $\tilde{m}_s^2 > 0.66$ (light), $\tilde{m}_s^2 > 0.75$ (white).

for $\Lambda_s = 10^{11}$ GeV. Thus the non-degeneracy of gauge couplings and gauginos contributes negatively to the bound even at the QFP. Second, away from the QFP, the bound asymptotes to the values with

$$\rho_p = \frac{1}{1 + 3.17\cos^2\overline{\beta}^{\mathrm{QFP}}} \sim 0.57\,. \tag{3.32}$$

However, the quantity multiplying $\tilde{M}_s^2$ in the bound is now $(\sigma_p - 1)$ which is a larger *negative* factor than $(\rho_p - 1)$.

We now further specialise to the mirage unification models with $V_0 = 0$, which have degenerate A-terms and degenerate scalar masses at the string scale;

$$\tilde{A}_s = -1\,, \qquad \tilde{m}_s^2 = \text{unconstrained}\,. \tag{3.33}$$

Contours of the LH_2, LLE bound are shown in figure 3, for varying $\tan\beta$ and Λ_s. The diagram shows that a lower string scale removes the dangerous minima. Indeed, for the WP model value of $\Lambda_s \sim 10^{11}$ GeV, there are *no* CCB minima appearing along the

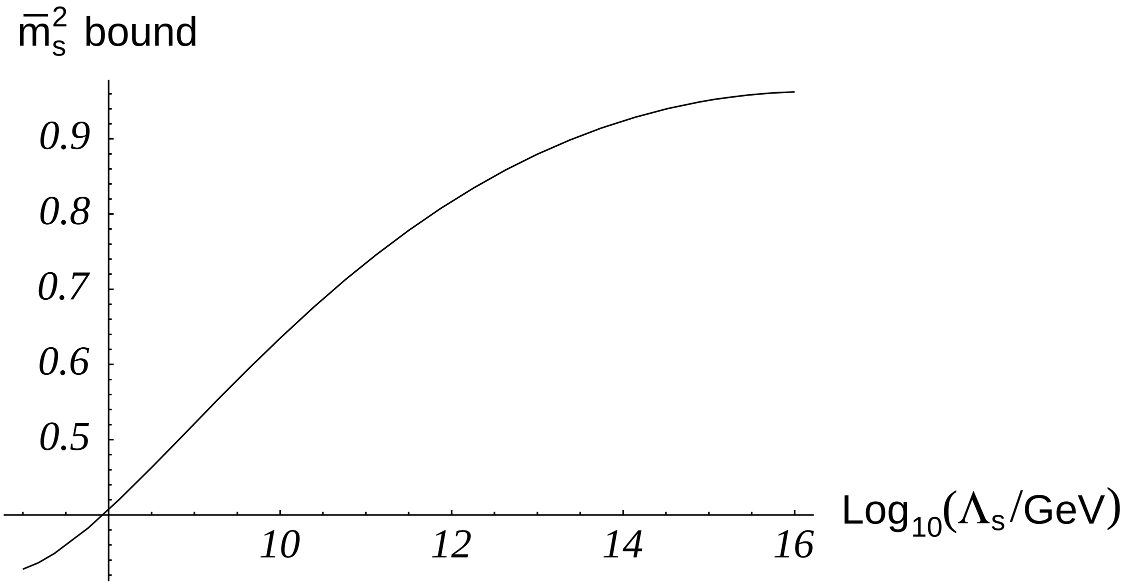

Figure 4: Charge and colour breaking bounds with a lower string scale at the quasi fixed point (QFP) for $\mu = 500\,\mathrm{GeV}$. The figure shows lower bounds on the string scale values of $\tilde{m}_s^2 = m_s^2/M_{1/2}^2$ for varying Λ_s.

LH_2, LLE direction except close to the QFP ($\tan\beta \lesssim 3$) or for negative scalar mass squared values ($m_s^2 < 0$). At the QFP we find that the bound at $\Lambda_s = 2\ 10^{16}\,\mathrm{GeV}$ is $\tilde{m}_s^2 \gtrsim 0.95$ but drops rapidly towards smaller values of Λ_s, as shown in figure 4. A full numerical determination of the bounds for specific points in parameter space is in accord with figures 3 and 4. It also shows that the bounds are in fact not overly sensitive to the precise values of α_1 and α_2 at Λ_s since the running is dominated by α_3.

Moreover, this behaviour is expected to be a general feature resulting from the low string scale pushing the CCB minimum to low scales. For example we can analyse the bound at large $\tan\beta$ where eq. (3.32) holds. Choosing $M_s^2 = 0$ and adjusting A_s to make $A_0 = M_{1/2}$, one finds that, away from the QFP, there are no CCB minima for any positive choice of non-universal mass-squared parameters at the string scale for $\Lambda_s \lesssim 10^{10}\,\mathrm{GeV}$. In other words, for these intermediate and low string scales one may always adjust A_s to remove CCB minima. Conversely, choosing a large enough value of A_s forms a CCB minimum at any Λ_s.

For $\Lambda_s \lesssim 10^7\,\mathrm{GeV}$ the analytic approximations we have been using break down for reasons outline above. Specifically, instead of evaluating the parameters at the renormalisation scale $\phi = h_t\langle h_2^0\rangle$, it is now more accurate to evaluate them at the scale $\phi = g_2\langle l\rangle$ (in the LLE, LH_2 direction) since this would be the largest relevant mass. Using this definition for ϕ we find numerically that minima do not reappear when Λ_s is lowered still further, as expected due to the dominance of the positive $m_{L_{ii}}^2 + m_{L_{33}}^2 + m_E^2$ contribution to the potential at low VEVs.

4. Summary

To summarise, we have examined constraints on the MSSM coming from the QFP scenario and CCB bounds when the string scale is lower than the canonical unification value of 10^{16-17} GeV. The quasi-fixed behaviour is weakened somewhat as the scale is reduced, i.e. weak MSSM parameters retain more information about their high energy boundary conditions. Very strict bounds upon the string scale are obtained from the LEP2 lower bound upon the lightest Higgs mass in the QFP scenario. Current limits exclude the QFP scenario for string scales between 10^6 and 10^{11} GeV for $m_t = 175 \pm 5$ GeV. This range of exclusion will increase by the end of running of LEP2, as the bounds improve. Run II of the Tevatron is expected to decrease the errors upon m_t significantly, with important implications for the range of Λ_s ruled out in the quasi-fixed scenario. For example, an error of 1 GeV upon m_t would rule out the QFP scenario for all $\Lambda_s > 10^5$ GeV.

CCB bounds also give important constraints upon the quasi-fixed scenario. We provided an analytic treatment of CCB bounds with lower string scales which we confirmed with a more accurate numerical check. It is clear from our results that lowering the string scale significantly weakens the CCB bounds. As an example, we considered the most restrictive case of the QFP. In this case the lower bound upon string-scale, degenerate, scalar mass-squared values $\tilde{m}_s^2$ is weakened by 30% in the WP model, $\Lambda_s = 10^{11}$ GeV. Remarkably, for $\tan\beta > 2$ and $\Lambda_s < 10^{10}$ GeV, the CCB bound is merely $m^2 > 0$ for *any* non-universal pattern of supersymmetry breaking.

Although we have concentrated on a particular subset of models (i.e. those that preserve the "GUT gaugino relation"), we argue that our conclusions are true in a more general case. As the string scale is lowered, provided that all mass-squared values are initially positive, the CCB minima are inevitably pushed to lower VEVs. At these low scales, the negative m_2^2 term no longer dominates the potential along the most dangerous F and D-flat directions.

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